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EVALUATION OF THE CONTACT TEMPERATURE AND WEAR OF A COMPOSITE FRICTION PAD IN BRAKING

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We suggest an analytical model to determine the contact temperature and wear of the working surface of the elements of friction brakes. It is assumed that one of the elements of the friction pair represents a two-period system of conjugated dissimilar layers and the other represents a homogeneous half-space. In the process of friction the wear factor depends linearly on contact temperature. We have studied the influence of the effective parameters of the composite and also of the parameter that characterizes the change in loading from zero to the nominal value and on the distribution of the contact temperature and wear in braking.

1. Statement of the Problem. According to the approach of [1, 2], to calculate the temperature and wear of the friction surface in braking we adopt the model represented in Fig. 1. It assumes that at the initial time $t = 0$ the friction pad, under the action of a normally distributed load of intensity P, is pressed against a steel disk. In general, the load increases monotonically from zero at $t = 0$ to the maximum value P_0 according to the law [3]

$$P(t) = P_0 P^*(t/t_m), \quad P^*(t) = 1 - \exp(-t). \tag{1}$$

Here the rate of braking V changes from the initial one V_0 at $t = 0$ to the zero one at the time of stopping $t = t_s$ in the following way [4]:

$$V(t) = V_0 V^*(\tau), \quad V^*(\tau) = 1 - \tau + \tau_m P^*(\tau^*), \quad 0 \le t \le t_s. \tag{2}$$

When $t_m \neq 0$, to calculate the time of stopping t_s we use the condition $V(t_s) = 0$, which, according to (2), leads to a nonlinear equation:

$$t_s - t_m P^*(t_s/t_m) = t_s^0. \tag{3}$$

Friction on the contact surface between the pad and the disk causes heat generation that leads to heating of the elements of the friction pair. The intensity of the frictional heat flux q is equal to the specific power of the friction forces [2] and is defined by the expression

$$q(t) = f(t)\,P(t)\,V(t), \quad 0 \le t \le t_s. \tag{4}$$

We assume that both bodies are elastic heat-conducting half-spaces (Fig. 1), i.e., the propagation of heat only along the z normal to the friction surface is taken into account [2]. In turn, the pad represents a composite that consists of a two-period system of dissimilar layers of thicknesses l_1 and l_2. The mechanical and thermal contacts between the layers of the composite are perfect. The quantities that relate to the pad are marked thereafter by the subscript p (pad) and those relating to the disk by the subscript d (disk).

The thermal problem of friction in braking presupposes the solution of the nonstationary heat conduction equations

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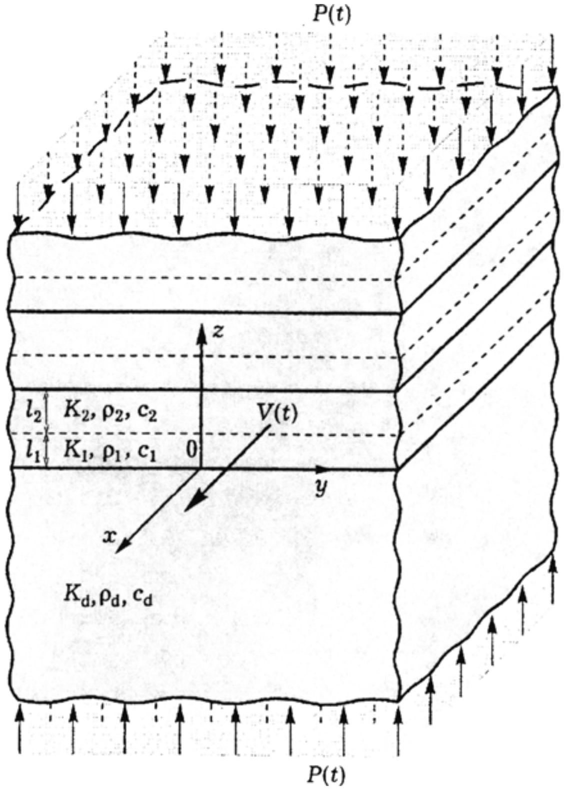

Fig. 1. Diagram of tribocontact.

$$\frac{\partial^2 T_i(z, t)}{\partial z^2} = \frac{1}{k_i} \frac{\partial T_i(z, t)}{\partial t}, \quad z > 0 \quad \text{for} \quad i = \text{p}, \quad z < 0 \quad \text{for} \quad i = \text{d}; \quad 0 \le t \le t_s, \tag{5}$$

subject to initial conditions

$$T_i(z, 0) = 0, \quad i = \text{p, d}, \tag{6}$$

conjugation conditions

$$T_\text{p}(0, t) = T_\text{d}(0, t) \equiv T(t), \quad 0 \le t \le t_s, \tag{7}$$

$$- K_\text{p} \frac{\partial T_\text{p}(z, t)}{\partial z}\bigg|_{z=0+} + K_\text{d} \frac{\partial T_\text{d}(z, t)}{\partial z}\bigg|_{z=0-} = q(t), \quad 0 \le t \le t_s, \tag{8}$$

and conditions of regularity

$$T_i \to 0, \quad i = \text{p, d} \quad \text{for} \quad |z| \to \infty, \quad 0 \le t \le t_s. \tag{9}$$

The effective coefficients of thermal conductivity K_p and thermal diffusivity k_p of the composite considered, which were obtained by the method of homogenization by means of microlocal parameters [5], have the form [6]:

$$K_\text{p} = K_1 \left(1 + \frac{[K]}{\eta K}\right), \quad k_\text{p} = \frac{K}{\widetilde{c}\widetilde{\rho}}, \quad K = \widetilde{K} - \frac{[K]^2}{\widehat{K}}, \quad \widehat{K} = \frac{K_1}{\eta} + \frac{K_2}{1 - \eta},$$
$$(\widetilde{K}, \widetilde{\rho}, \widetilde{c}) = \eta (K_1, \rho_1, c_1) + (1 - \eta)(K_2, \rho_2, c_2), \quad [K] = K_2 - K_1. \tag{10}$$

The pad is the weaker element (as regards thermal effect) of the friction pair; its friction-wear properties change significantly with an increase in temperature [7]. Therefore, for the wear factor of the working surface of the pad we will adopt a linear dependence on the contact temperature T:

$$m(t) = m_0 + m_1 T(t), \tag{11}$$

which is sufficiently substantiated at small temperature gradients [8].

Then, we calculate the wear of the pad from the formula [9]

$$I(t) = \int_0^1 m(T)\, q(t_0)\, dt_0, \quad 0 \le t \le t_s, \tag{12}$$

where the intensity of the frictional heat flux q is determined from formulas (1), (2), and (4).

2. Temperature. The solution of the boundary-value problem of heat conduction (5)-(9) obtained by applying the Laplace integral transform with respect to time t is represented in the form of a convolution [10]:

$$T_i(z, t) = \Lambda_0 \Lambda^* \int_0^{\tau} P^* [(\tau - \tau_0)/\tau_m]\, V^* (\tau - \tau_0)\, \tau_0^{-1/2} \exp(-\zeta_i^2/\tau_0)\, d\tau_0, \tag{13}$$

$$i = \mathrm{p}, \mathrm{d}, \quad 0 \le \tau \le \tau_s,$$

where

$$\Lambda^* = \frac{1}{1 + k_\varepsilon}; \quad k_\varepsilon = \frac{K_p}{K_d}\sqrt{\left(\frac{k_d}{k_p}\right)}; \quad \zeta_i = \frac{|z|}{2\sqrt{k_i t_s^0}}, \quad i = \mathrm{p}, \mathrm{d};$$
$$\Lambda_0 = \frac{f_0 P_0 V_0}{K_d}\sqrt{\left(\frac{k_d t_s^0}{\pi}\right)}; \quad \tau = \frac{t}{t_s^0}; \quad \tau_m = \frac{t_m}{t_s^0}; \quad \tau_s = \frac{t_s}{t_s^0}. \tag{14}$$

Substitution of the functions P^* (1) and V^* (2) under the sign of the integral in relation (13) and subsequent integration at $\zeta_i = 0$ make it possible to find the temperature on the friction surface:

$$T(t) = \Lambda_0 \Lambda^* T^*(t), \quad 0 \le t \le t_s, \tag{15}$$

where

$$T^*(t) = \left(2 + \tau_m - \frac{4}{3}\tau\right)\sqrt{\tau} - \left(1 + \frac{3}{2}\tau_m - \tau\right) 2\sqrt{\tau_m}\, F(\sqrt{\tau^*}) + \tau_m \sqrt{\tau_m}\, F(\sqrt{2\tau^*}), \tag{16}$$

$F(\tau) = \exp(-\tau^2)\int_0^{\tau} \exp(x^2)dx$ is the Dawson integral, to calculate the values of which we use the formulas [11]

$$F(\tau) = \sum_{i=0}^{\infty} \frac{(-2\tau^2)^i}{(2i+1)!!}, \quad 0 \le \tau \le 3, \quad F(\tau) = \sum_{i=0}^{n} \frac{(2i-1)!!}{(2\tau^2)^{i+1}}, \quad \tau > 3,$$

where $(-1)!! = 1$.

When $t_m = 0$ ($t_s = t_s^0$), to calculate the contact temperature in braking with constant deceleration [12], relation (15) yields the expression

$$T(t) = 2\Lambda_0\Lambda^* \left(1 - \frac{2}{3}\frac{t}{t_s}\right) \sqrt{\left(\frac{t}{t_s}\right)}, \quad 0 \le t \le t_s .$$

3. Wear. Having substituted formula (15) for the contact temperature T in (12) and integrating, we obtain a formula for the wear of the rubbing surface of the friction pad:

$$I(t) = m_0^* I_0(t) + m_1^* \Lambda_0 \Lambda^* I_1(t), \quad 0 \le t \le t_s, \tag{17}$$

where

$$I_0(t) = \tau - \tau^2/2 + \tau_m (\tau - \tau_m - 1) P^*(\tau^*) + \tau_m^2 P^*(2\tau^*)/2, \tag{18}$$

$$I_1(t) = I_1^{(1)}(t) + I_1^{(2)}(t) + I_1^{(3)}(t), \tag{19}$$

$$I_1^{(1)}(t) = \frac{2}{3}(1 + \tau_m)(2 + \tau_m)\tau\sqrt{\tau} - \frac{2}{15}(10 + 7\tau_m)\tau^2\sqrt{\tau} + \frac{8}{21}\tau^3\sqrt{\tau} -$$

$$- (1 + 2\tau_m)(2 + \tau_m)\tau_m\sqrt{\tau_m}\left[\frac{1}{2}\sqrt{\pi}\,\mathrm{erf}(\sqrt{\tau^*}) - \tau^*\exp(-\tau^*)\right] + \frac{1}{3}(10 +$$

$$+ 11\tau_m)\tau_m^2\sqrt{\tau_m}\left[\frac{3}{4}\sqrt{\pi}\,\mathrm{erf}(\sqrt{\tau^*}) - \sqrt{\tau^*}\left(\frac{3}{2} + \tau^*\right)\exp(-\tau^*)\right] -$$

$$- \frac{4}{3}\tau_m^3\sqrt{\tau_m}\left[\frac{15}{8}\sqrt{\pi}\,\mathrm{erf}(\sqrt{\tau^*}) - \sqrt{\tau^*}\left(\frac{15}{4} + \frac{5}{2}\tau^* + \tau^{*2}\right)\exp(-\tau^*)\right] +$$

$$+ \frac{1}{2}(2 + \tau_m)\tau_m^2\sqrt{\tau_m}\left[\frac{1}{2}\sqrt{\left(\frac{\pi}{2}\right)}\,\mathrm{erf}(\sqrt{2\tau^*}) - \sqrt{\tau^*}\exp(-\tau^*)\right] -$$

$$- \frac{2}{3}\tau_m^3\sqrt{\tau_m}\left[\frac{3}{8}\sqrt{\left(\frac{\pi}{2}\right)}\,\mathrm{erf}(\sqrt{2\tau^*}) - \sqrt{\tau^*}\left(\frac{3}{8} + \tau^*\right)\exp(-\tau^*)\right],$$

$$I_1^{(2)}(t) = 2\sqrt{\tau_m}\left[\left(2 + \frac{5}{2}\tau_m\right)M_{101}(\tau) - (1 + \tau_m)\left(1 + \frac{3}{2}\tau_m\right)M_{001}(\tau) - M_{201}(\tau) +\right.$$

$$+ M_{211}(\tau) - \left(2 + \frac{7}{2}\tau_m\right)M_{111}(\tau) + (1 + 2\tau_m)\left(1 + \frac{3}{2}\tau_m\right)M_{011}(\tau) + \tau_m M_{121}(\tau) -$$

$$\left. - \tau_m\left(1 + \frac{3}{2}\tau_m\right)M_{021}(\tau)\right],$$

$$I_1^{(3)}(t) = \tau_m\sqrt{2\tau_m}\,[(1 + \tau_m)M_{002}(\tau) - M_{102}(\tau) + M_{112}(\tau) -$$

$$- (1 + 2\tau_m)M_{012}(\tau) + \tau_m M_{022}(\tau)],$$

$$M_{001}(\tau) = \tau_m[\sqrt{\tau^*} - F(\sqrt{\tau^*})],$$

$$M_{101}(\tau) = \tau_m^2\left[\sqrt{\tau^*} + \frac{1}{3}\tau^*\sqrt{\tau^*} - (1 + \tau^*)F(\sqrt{\tau^*})\right],$$

$$M_{201}(\tau) = \tau_m^3\left[2\sqrt{\tau^*} + \frac{2}{3}\tau^*\sqrt{\tau^*} + \frac{1}{5}\tau^{*2}\sqrt{\tau^*} - (2 + 2\tau^* + \tau^{*2})F(\sqrt{\tau^*})\right],$$

$$M_{011}(\tau) = \frac{\tau_m}{2}\left[\mathrm{erf}\,(\sqrt{\tau^*}) - \exp(-\tau^*)\,F(\sqrt{\tau^*})\right],$$

$$M_{111}(\tau) = \frac{\tau_m^2}{2}\left[\mathrm{erf}\,(\sqrt{\tau^*}) - \frac{1}{2}\sqrt{\tau^*}\exp(-\tau^*) - \left(\frac{1}{2}+\tau^*\right)\exp(-\tau^*)\,F(\sqrt{\tau^*})\right],$$

$$M_{211}(\tau) = \frac{\tau_m^3}{2}\left[\frac{7}{4}\mathrm{erf}\,(\sqrt{\tau^*}) - \frac{1}{2}\sqrt{\tau^*}\left(\frac{5}{2}+\tau^*\right)\exp(-\tau^*) - \tau^{*2}\exp(-\tau^*)\,F(\sqrt{\tau^*}) - \right.$$
$$\left. - \left(\frac{1}{2}+\tau^*\right)\exp(-\tau^*)\,F\sqrt{\tau^*}\right],$$

$$M_{021}(\tau) = \frac{\tau_m}{3}\left[\frac{1}{2}\sqrt{\left(\frac{\pi}{2}\right)}\,\mathrm{erf}\,(\sqrt{2\tau^*}) - \exp(-2\tau^*)\,F(\sqrt{\tau^*})\right],$$

$$M_{121}(\tau) = \frac{\tau_m^2}{2}\left[\frac{1}{2}\sqrt{\left(\frac{\pi}{2}\right)}\,\mathrm{erf}\,(\sqrt{2\tau^*}) - \frac{1}{4}\sqrt{\tau^*}\exp(-2\tau^*) - \left(\frac{1}{3}+\tau^*\right)\exp(-2\tau^*)\,F(\sqrt{\tau^*})\right],$$

$$M_{002}(\tau) = \tau_m\left[\sqrt{\left(\frac{\tau^*}{2}\right)} - \frac{1}{2}F(\sqrt{2\tau^*})\right],$$

$$M_{102}(\tau) = \tau_m^2\left[\frac{1}{2}\sqrt{\left(\frac{\tau^*}{2}\right)} + \frac{\tau^*}{3}\sqrt{\left(\frac{\tau^*}{2}\right)} - \frac{1}{2}\left(\frac{1}{2}+\tau^*\right)F(\sqrt{2\tau^*})\right],$$

$$M_{112}(\tau) = \tau_m^2\left[\frac{5}{9\sqrt{2}}\mathrm{erf}\,(\sqrt{\tau^*}) - \frac{1}{3}\sqrt{\left(\frac{\tau^*}{2}\right)}\exp(-\tau^*) - \frac{1}{3}\left(\frac{1}{3}+\tau^*\right)\exp(-\tau^*)\,F(\sqrt{2\tau^*})\right],$$

$$M_{012}(\tau) = \tau_m\left[\frac{2}{3\sqrt{2}}\mathrm{erf}\,(\sqrt{\tau^*}) - \frac{1}{3}\exp(-\tau^*)\,F(\sqrt{2\tau^*})\right],$$

$$M_{022}(\tau) = \tau_m\left[\frac{\sqrt{\pi}}{8}\mathrm{erf}\,(\sqrt{2\tau^*}) - \frac{1}{4}\exp(-2\tau^*)\,F(\sqrt{2\tau^*})\right],$$

$$m_0^* = m_0 f_0 V_0 P_0 t_s^0,\quad m_1^* = m_1 f_0 V_0 P_0 t_s^0.$$

4. Numerical Analysis. We performed calculations by the following scheme: we assigned the dimensionless duration τ_m of the increase in loading from zero to the maximum value P_0. Using Eq. (3), we determined the dimensionless braking time τ_s. The dimensionless functions $T^*(t)$, $I_0(t)$, and $I_1(t)$ were determined from formulas (14), (15), (17), and (18), respectively.

The function $T^*(t)$ (the dimensionless contact temperature) reaches its maximum in braking with constant deceleration (Fig. 2). Its distribution in braking is characterized by appreciable nonuniformity. Increasing rapidly at the initial time instants, $T^*(t)$ attains the maximum value and decreases as $t \to t_s$. As the parameter τ_m increases, the time needed for the function $T^*(t)$ to reach the maximum value shifts to the direction of complete stopping.

The dimensionless function $I_0(t)$ (18) characterizes the wear of the working surface of the pad in the absence of frictional heat generation. It attains its maximum value at the time of stopping (Fig. 3a). Within the time interval $0 \le t \le t_s^0$ the greatest wear occurs in braking with constant deceleration. At the time of stopping the wear does not depend on the parameter τ_m.

The function $I_1(T)$ (19) is also a monotone increasing function (Fig. 3b), with its maximum value being reached at the time of stopping at $t = t_s$, but it substantially depends on the parameter τ_m. For a fixed braking time

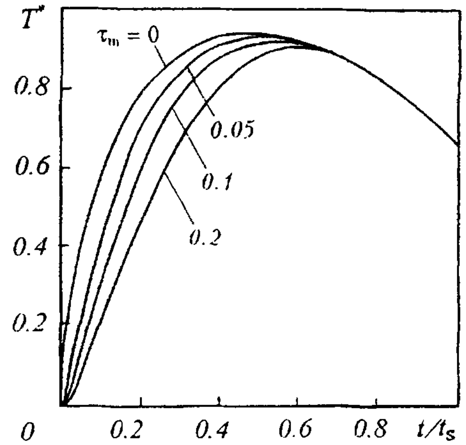

Fig. 2. Change in dimensionless contact temperature $T^* = T/(\Lambda_0\Lambda^*)$ in braking with different parameters τ_m.

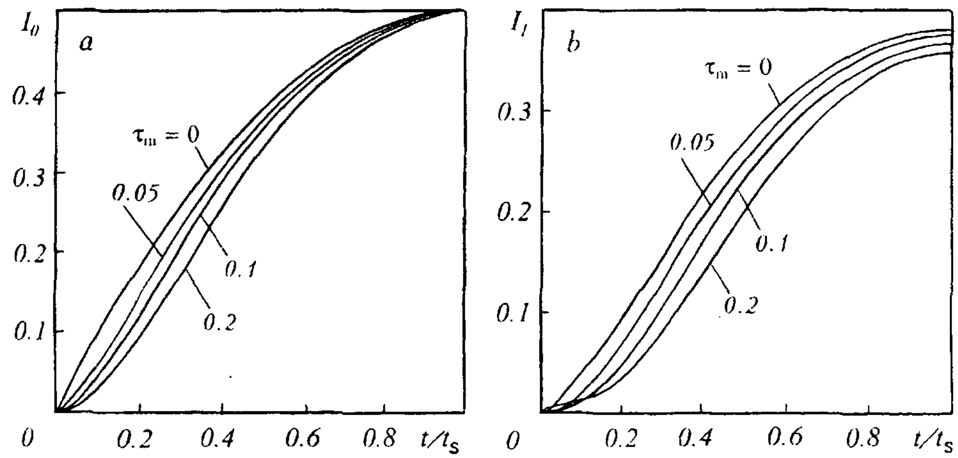

Fig. 3. Change in functions I_0 (a) and I_1 (b) in braking with different parameters τ_m.

the value of $I_1(t)$ is the smallest in the case of uniformly decelerated braking. An increase in the nonuniformity of the change in the rate in the process of braking (an increase in the parameter τ_m) leads to an increase in $I_1(t)$.

The effect of the thermophysical and geometric parameters of the composite manifests itself by means of the factor Λ^* (13). The coefficient k_ε in Λ^* characterizes the thermal activity of the pad relative to the disk [10]. Taking into consideration formulas (10), we represent k_ε in the form

$$k_\varepsilon = k_\varepsilon^* \, g_p(K^*) \sqrt{\left(\frac{\tilde{g}(\rho^*)\,\tilde{g}(c^*)}{g(K^*)}\right)}\,, \tag{20}$$

where

$$K^* = \frac{K_2}{K_1}\,; \quad \rho^* = \frac{\rho_2}{\rho_1}\,; \quad c^* = \frac{c_2}{c_1}\,; \quad k_\varepsilon^* = \sqrt{\left(\frac{K_1\,\rho_1 c_1}{K_d\,\rho_d c_d}\right)}\,.$$

On the basis of formulas (10) the functions of the effect in relation (20) have the form

$$g(x) = \tilde{g}(x) - \frac{[g(x)]^2}{\hat{g}(x)}\,, \quad g_p(x) = 1 + \frac{[g(x)]}{\eta\,\hat{g}(x)}\,,$$

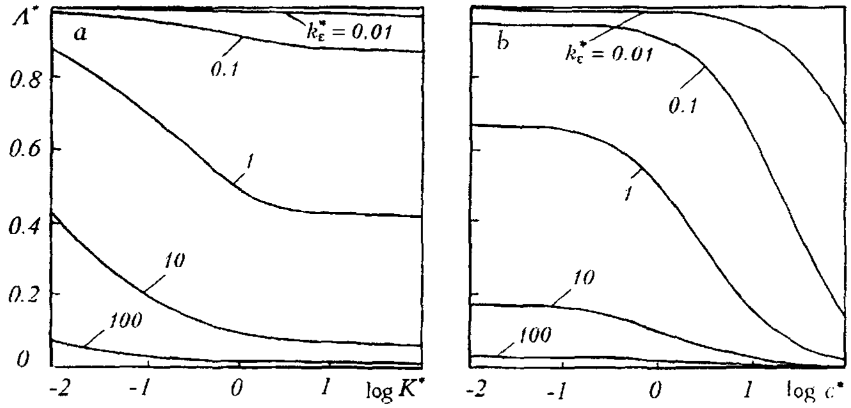

Fig. 4. Dimensionless function Λ^* vs relative thermal conductivity of composite K^* (a) ($\eta = 0.5$; $\rho^* = 1$; $c^* = 1$) and vs relative specific heat capacity of composite c^* (b) ($\eta = 0.5$; $\rho^* = 1$; $K^* = 1$) for different values of dimensionless parameter k_ε^*.

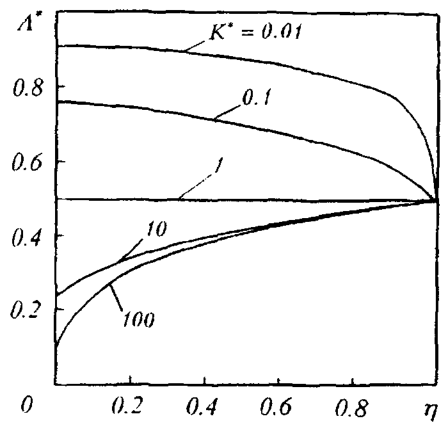

Fig. 5. Dimensionless function Λ^* vs relative thickness of layers of composite η at $k_\varepsilon^* = 1$; $\rho^* = 1$; $c^* = 1$ for different values of dimensionless parameter K^*.

$$\tilde{g}(x) = \eta + (1 - \eta)\, x\,, \quad \hat{g}(x) = \frac{1}{\eta} + \frac{x}{1 - \eta}\,, \quad [g(x)] = x - 1\,.$$

Thus, the input parameters in calculation of Λ^* are K^*, ρ^*, c^*, η and k_ε^*.

An increase in the relative coefficient of thermal conductivity K^* leads to a decrease in Λ^* (Fig. 4a). Similar behavior of Λ^* is observed with an increase in the relative specific heat capacity c^* (Fig. 4b). As the thickness l_1 of one of the components of the composite increases, the factor Λ^* decreases (increases) for $K^* < 1$ ($K^* > 1$) (Fig. 5.).

Since the temperature (15) and wear (17) are linearly related to Λ^*, the character of their dependence on the effective properties of the composite is determined by the behavior of this factor.

CONCLUSIONS

1) An increase in the content of the component with the higher coefficient of thermal conductivity ($K_1 > K_2$) in a periodic cell of a composite pad leads to reduction of the contact temperature. Conversely, the temperature on the friction surface in braking will be the highest at a greater content of heat insulator (or of a material with a low coefficient of thermal conductivity).

2) An increase in the thickness of the composite with the higher coefficient of specific heat capacity ($c_1 > c_2$) also leads to reduction of the contact temperature.

Thus, to reduce the temperature in the contact region in braking, it is necessary to increase the thickness of the component with the higher coefficient of thermal conductivity and specific heat in the composite pad.

NOTATION

t, time; z, axial coordinate; T, temperature; V, slipping velocity; V_0, initial rate of braking; P, pressure; P_0, maximum pressure; q, specific power of friction forces; K_p, K_d, k_p, k_d, coefficients of thermal conductivity and thermal diffusivity of the pad and disk, respectively; l_1 and l_2, thicknesses of dissimilar layers of composite; $l = l_1 + l_2$, thickness of periodically repeated cell of composite; K_1, K_2, coefficients of thermal conductivity of components of composite; c_1, c_2, specific heats of components of composite; ρ_1, ρ_2, density of materials of components of composite; K^*, relative thermal conductivity of composite; c^*, relative specific heat of composite; $\eta = l_1/l$, relative thickness of layers of composite; T^*, dimensionless contact temperature; f_0, friction factor; m, wear factor; t_m, time needed for loading to attain maximum value; t_s, time of stopping; $t_s^0 = 2W/(fP_0V_0)$, time of braking in the case of instantaneous ($t_m = 0$) attainment of the nominal value P_0 by loading; $\tau^* = t/t_m$; W, reduced kinetic energy at beginning of braking; erfc $(\cdot) = 1 -$ erf $(\cdot)$; erf $(\cdot)$, probability integral.

REFERENCES

1. A. V. Chichinadze, *Calculation and Investigation of External Friction in Braking* [in Russian], Moscow (1967).
2. A. V. Chichinadze, É. D. Braun, A. G. Ginzburg, and Z. V. Ignat'eva, *Calculation and Investigation of External Friction in Braking* [in Russian], Moscow (1979).
3. A. G. Ginzburg, in: *Optimum Use of Friction Materials in Friction Units of Machines* [in Russian], Moscow (1973), pp. 93-105.
4. E. V. Kovalenko, A. A. Evtushenko, and E. G. Ivanik, *Trenie Iznos*, **17**, No. 4, 435-441 (1996).
5. C. Wozniak, *Int. J. Eng. Sci.*, **25**, 483-499 (1987).
6. A. A. Yevtushenko, S. J. Matysiak, and E. G. Ivanyk, *Int. J. Heat Mass Transfer*, **40**, No. 9, 2115-2122 (1997).
7. M. P. Aleksandrov and A. L. Nosko, *Trenie Iznos*, **14**, No. 5, 895-901 (1993).
8. G. K. Annakulova, *Izv. Akad. Nauk UzSSR, Ser. Tekh. Nauk*, 21-26 (1989).
9. A. G. Ginzburg and A. V. Chichinadze, in: *Friction and Wear of Friction Materials* [in Russian], Moscow (1977), pp. 26-30.
10. A. V. Lykov (A. V. Luikov), *Heat Conduction Theory* [in Russian], Moscow (1967).
11. J. R. Barber and C. J. Martin-Moran, *Wear*, **79**, 11-19 (1982).
12. G. A. G. Fazekas, *SAE Trans.*, No. 1, 279-284 (1953),